THE SCIENCE REVISION WEEKEND

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S381
The Energetic Universe

Block 2
Nucleosynthesis and Stellar Remnants

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S381 Block 2 – Nucleosynthesis and Stellar Remnants

Post-main-sequence evolution

- Outcome depends critically on mass of the star.
- Mass determines maximum core temperature reached during various evolutionary phases.
- Mass determines how far the nuclear burning processes can go in terms of producing heavier elements.
- Dramatic difference in fate between stars with:

  $M < 8\ \mathrm{M_{Sun}}$ which become white dwarfs.

  $M > 8\ \mathrm{M_{Sun}}$ which explode as supernovae (SN II).

Evolution of low-mass stars $0.5\ \mathrm{M_{Sun}} < M < 8\ \mathrm{M_{Sun}}$

- Core H-burning ceases when core H is depleted.
- H burning continues in a shell around the core.
- Core contracts and heats up.
- Simultaneously, the envelope expands and cools.
- Initially, little change in luminosity; radius increases and the temperature drops to compensate. The star moves to the right across the HR diagram, where it is classed as a red giant.
- Luminosity then starts to increase.
- Core helium burning starts when the core temperature reaches 10^8 K.

He burning: the triple-alpha reaction B2,§6.3

There is no stable mass-8 nucleus, so He burning cannot proceed simply by $^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{X}+\mathrm{Y}$.

Require three helium nuclei (i.e. alpha-particles) so that the third one may be captured by the unstable 8X nucleus before it decays. The reaction pathway is

$^{4}\mathrm{He} + {}^{4}\mathrm{He} \Leftrightarrow {}^{8}\mathrm{Be}$ $\quad$ $^{4}\mathrm{He} + {}^{8}\mathrm{Be} \Leftrightarrow {}^{12}\mathrm{C}^{*}$ $\quad$ $^{12}\mathrm{C}^{*} \rightarrow {}^{12}\mathrm{C}+2\gamma$

The double-headed arrows indicate that the first two stages are reversible reactions which can (and do) proceed in both directions. Only the final decay of the excited state $^{12}\mathrm{C}^{*}$ to the ground-state $^{12}\mathrm{C}$ is irreversible in practice. $\mathrm{d}n_{12}/\mathrm{d}t = n_{12^{*}}/\tau({}^{12}\mathrm{C}^{*}\rightarrow{}^{12}\mathrm{C})$

The existence of an equilibrium between forward and backward reactions means that the abundances of $^{4}\mathrm{He}$, $^{8}\mathrm{Be}$, and $^{12}\mathrm{C}^{*}$ can be calculated from the chemical potential

$$\mu = mc^{2} - kT \ln (g_{s} n_{\mathrm{Q,NR}}/n)$$ B2,§6.2

since $\mu({}^{4}\mathrm{He}) + \mu({}^{4}\mathrm{He}) = \mu({}^{8}\mathrm{Be})$

and $\mu({}^{4}\mathrm{He}) + \mu({}^{8}\mathrm{Be}) = \mu({}^{12}\mathrm{C}^{*})$

The reactions have sizeable Coulomb barriers, and hence large Gamow energies and Gamow peaks, so require much larger temperatures than H burning: $T_{3\alpha} \sim 10^{8}$ K.

The fractional mass defect for He burning is 0.0007, about 1/10 of the value for H-burning, with the consequence that He-burning can provide at most 10% of the energy as H burning. B2,§6.3.2

This means the He-burning lifetime of a star is at most ~1/10 of the main-sequence lifetime.

The helium flash in low-mass stars

$$n_e/n_{Q,NR} \propto R^{-3/2}$$

B2,§6

As the core contracts, the gas approaches degeneracy.

Major difference between core-He ignition depending on mass:

- At $M < 2.5\ M_{Sun}$, the core is degenerate when T_c reaches 10^8 K for He ignition.

  In degenerate gas, temperature rises without causing an increase in pressure.

  Due to high temperature sensitivity on the He-burning reaction, a thermonuclear runaway occurs with a huge release of energy.

- At $M > 2.5\ M_{Sun}$, the core is not degenerate when T_c reaches 10^8 K for He ignition.

  In non-degenerate gas, temperature rise causes an increase in pressure, and the core expands slightly to lower the temperature again, and hence to lower the reaction rate until it just balances energy loses from the stellar surface.

Despite the energetic nature of the He flash, the event is not apparent at the surface of the star as changes in T or L for several thousand years. (It takes this long for the energy released in the flash to reach the surface.)

Core-He-burning evolution of low-mass stars

Location of core-He-burning stars depend on mass and metallicity.

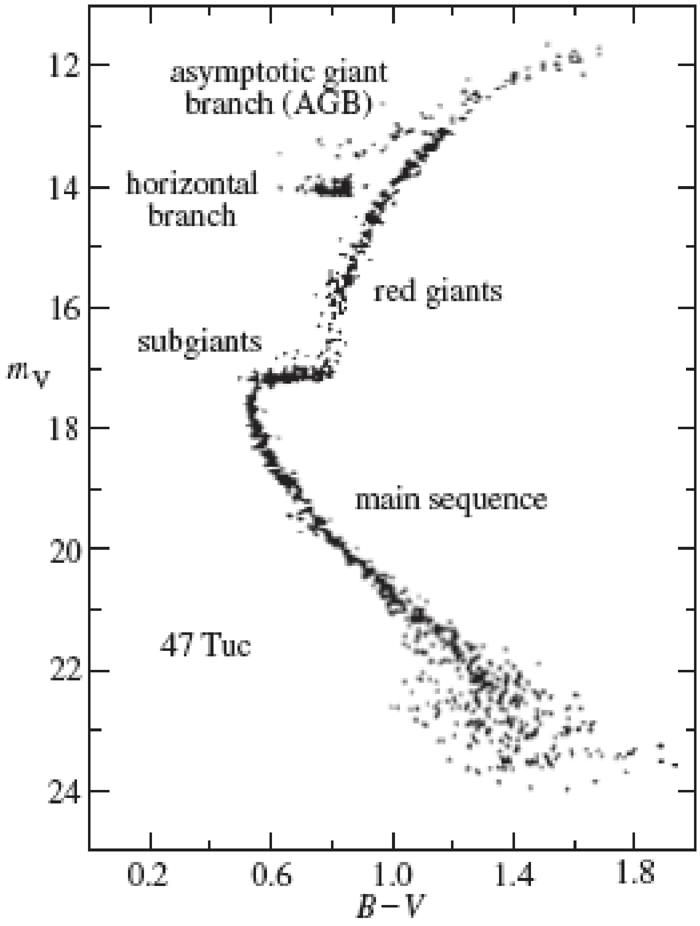
Low-mass stars become either:

- Red-giant clump stars if metallicity ~ Z_{Sun}
- Horizontal-branch stars if metallicity ~ Z_{PopII}

Red-giant clump stars are just barely to the left of the giant branch, but only halfway up from the base of the RGB to the tip.

Horizontal-branch stars are considerably blueward of the RGB, and may be in the instability strip as RR Lyrae stars.

A high-mass star begins a blue-loop to the left in the HR diagram.



B2 Fig. 43. Observational H–R diagram of the old globular cluster 47 Tuc, showing a clear asymptotic giant branch (AGB) occupied by shell-He-burning stars making their way from the horizontal branch toward the top of the giant branch.

Post-core-He-burning evolution of low-mass stars

B2,§6.7

Once core He becomes depleted, shell He-burning starts.
$\varepsilon_{3\alpha} \propto T^{30}$, so He-burning shell is very thin.

The star returns toward the giant branch, and evolves again to higher luminosity along the asymptotic giant branch (AGB).

It moves up and to the right in the HR diagram, and becomes very large and luminous. Mass loss increases greatly.

- Early (E-)AGB energy generation dominated by He shell.
- Subsequent shell-He burning becomes unstable.
- Gives rise to He-shell flashes:
  - Violent and repetitive releases of energy via He-burning that switches on and off.
  - Called thermally-pulsing (TP-)AGB phase.
- He pulse may last only 10^2 yr.
- Followed by 10^4 yr with just the H-burning shell operative.

Shell flashes may drive mass ejection that form planetary nebulae.

He-exhausted core contracts further, and core temperature rises.

Remaining He and the fresh C are burnt to oxygen by

$$^{12}_{6}\mathrm{C} + ^{4}_{2}\mathrm{He} \rightarrow ^{16}_{8}\mathrm{O} + \gamma$$

If $M < 8\ \mathrm{M_{Sun}}$, this is the most advanced reaction that occurs.

If $M > 8\ \mathrm{M_{Sun}}$, carbon burning becomes possible.

S-process reactions form heavier nuclei B2,§8.1

S-process reactions occur during shell-He burning phase of thermally-pulsing (TP)-AGB star.

S-process reactions are “slow” neutron captures onto existing seed nuclei (especially iron) to form heavier nuclei, especially those heavier than iron.

Neutron-capture rate is slow enough that unstable, neutron-rich isotopes have time to beta-decay before they capture another neutron.

S-process therefore follows path close to valley of beta-stability in chart of nuclides.

Mass loss and the planetary nebula phase

Part of the HR diagram corresponds to conditions in which the star will pulsate – the instability strip.

As a giant evolves to the left and right in the HR diagram, it will spend some time in the instability strip. Pulsational instabilities and/or He shell flashes may drive major mass loss at these times.

The outer envelope of the star is blown away, leaving a dense hot core which is a white dwarf.

Once core exposed enough to have $T_{\mathrm{eff}} > 10{,}000$ K, sufficient UV photons to ionise ejected envelope and produce planetary nebula.

White dwarf mass limit B2,§7.2

X_e = electron mass fraction
Y_e = number of electrons per nucleon $Y_e(\mathrm{H})=1$, $Y_e(\mathrm{He})=0.5$
$Y_e \approx (1+X_H)/2$

Equations of state for electron-degenerate gas:

- Non-relativistic gas $P_{NR} = K_{NR} n_e^{5/3}$ Ph,Eq5.10
- Ultra-relativistic gas $P_{UR} = K_{UR} n_e^{4/3}$ Ph,Eq5.11

Constants: $K_{NR} = \frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}$ $K_{UR} = \frac{hc}{4}\left(\frac{3}{8\pi}\right)^{1/3}$

Clayton stellar model $P_C \approx (\pi/36)^{1/3} G M^{2/3} \rho_C^{4/3}$ Ph,Eq.5.33

- Ultra-relativistic degenerate electrons cannot support a star.
- Degenerate electrons become ultra-relativistic at 1.4 M_{Sun}.
- Maximum supportable mass: Chandrasekhar mass.

White dwarf mass-radius relation B2,§7.2.2

- Small: radius ≈ a few thousand km.
- More-massive white dwarfs have smaller radii:

$$R_{WD} = \frac{R_{Sun}}{74}\left(\frac{M}{M_{Sun}}\right)^{-1/3}$$

- Radius does not depend on T.
- Does not expand or contract as it cools.
- Simply fades at constant radius.
- Results from degenerate state of WD matter.

Post-main sequence evolution of high-mass stars $M > 8\ \mathrm{M_{Sun}}$

Recap main-sequence phase:

- Very high luminosities
- Very short lifetimes (few million years)
- High core temperatures
  - H burning is via CNO cycle
  - Main pressure support in core is from radiation

After core H is depleted, H burning starts in a shell (as for low-mass stars), and the envelope expands.

Because of high mass of star, core temperature continues to increase after each successive nuclear burning phase until most of the core has been converted to iron – the most stable nuclide.

The sequence of nucleosynthesis

- Carbon-burning (at 5-9 × 10^8 K) on timescale ≈ 600 yr:
  $^{12}C(^{12}C,\alpha)^{20}Ne$ $^{12}C(^{12}C,p)^{23}Na$ $^{12}C(^{12}C,n)^{24}Mg$
- Neon-burning (at 1-1.7 × 10^9 K) on timescale ≈1 yr:
  $^{20}Ne(\gamma,\alpha)^{16}O$ and $^{20}Ne(\alpha,\gamma)^{24}Mg$
- Oxygen-burning (at 2-2.3 × 10^9 K) on timescale ≈ 6 months:
  $^{16}O(^{16}O,\alpha)^{28}Si$
- Silicon-burning (at 3-4 × 10^9 K) on timescale ≈1 day:
  $^{28}Si(\gamma,\alpha)^{24}Mg$

Elements between Si and the iron group in an equilibrium (NSE) between photodissociation and α-, p- and n-capture reactions.

$^{28}Si(\alpha,\gamma)^{32}S(\alpha,\gamma)^{36}Ar(\alpha,\gamma)^{40}Ca(\alpha,\gamma)^{44}Ti(\alpha,\gamma)\,^{48}Cr(\alpha,\gamma)^{52}Fe(\alpha,\gamma)^{56}Ni$.

For nuclear statistical equilibrium (NSE) relative abundances of iron-peak nuclei (Ti-Zn) governed by their binding energies:

$$n_{\mathrm{A}} n_{\alpha} / n_{(\mathrm{A}+\alpha)} \propto \exp(-Q/kT)$$

where Q is the energy released in the α-capture.

Once T_c reaches 7 × 10^9 K, most of core is converted to Fe:

- Iron the most stable nuclide.
- Energy can not be released from nuclear reactions involving iron, i.e. once a star has an Fe core it has run out of fuel.
- Fuel needed to provide pressure support against gravity.
- The star *rapidly* progresses to become a supernova (SN).

Core-collapse supernovae

B2,§8.2

- Electron degeneracy cannot support a core exceeding 1.4 M_{Sun} as core is too large.
- Core collapse begins
- Core temperature rises
- Once T_c reaches 10^{10} K, thermal γ rays photodissociate (break-up) Fe nuclei into α particles and neutrons.
- This absorbs energy, so the pressure drops further.
- As the density increases, so the does the energy of the degenerate electrons.
- Electron-capture reactions become energetically possible and in fact preferred:

$$e^- + p \rightarrow n + \nu \;=\; \text{neutronization}$$

- As electrons are consumed, the electron-degeneracy pressure drops, and the pressure, already inadequate, drops further.
- Core collapse accelerates until it is abruptly halted by the onset of neutron degeneracy.
- Density of ~ 3×10^{17} kg m^{-3} and $T_c \sim 10^{12}$ K.
- Core collapse stops abruptly with a slight rebound.

The exact mechanism for the supernova explosion is not yet clear, but three possibilities are:

- Shock-front propagates outwards through the infalling envelope and expels it.
- Shock-front heats the envelope to 10^{10} K, triggering nuclear burning in the envelope.
- Huge flux of neutrinos may heat and expel the envelope.

Energy released in supernova

Huge amount of energy is released ~10^{46} J:

• 99%	carried by neutrinos
• 1%	kinetic energy of ejected material
• 0.01%	radiation

Nucleosynthesis in the supernova

- The iron core is lost to a neutron star or black hole.
- Envelope temperature high enough for wide range of thermonuclear reactions to occur, producing elements up to the iron peak.
- Vast numbers of neutrons are produced as the core collapses, giving rise to neutron-capture reactions via the r-process.
- Supernovae are the main source of enrichment of elements heavier than iron in the interstellar medium (ISM), and of many radioactive isotopes such as:
  - ^{56}Ni (whose decay via ^{56}Co to ^{56}Fe is evident in supernova light curves).
  - ^{26}Al (important for radiogenic heating in planets).

Note: Supernovae are not the source of some of the most abundant heavy elements in the ISM; carbon and nitrogen are mostly produced in low-mass stars.

The r-process

B2,§8.1

- R-process stands for “rapid” neutron-capture process.
- Unstable, neutron-rich nuclides do not have time to beta-decay before the next neutron capture, so the r-process can build elements that the s-process cannot.
- When neutron source ends, highly neutron-rich isotopes beta-decay back towards the valley of beta-stability. Once a stable nuclide is formed, no more decays take place.
    - This shields some isotopes from having an r-process contribution; such isotopes have only an s-process contribution.
    - It also allows the formation, by the r-process, of isotopes that are too far from the valley of beta-stability for the s-process to reach them.
- Most isotopes, however, have contributions from both the s- and r-process.
- Nuclei produced on the s- or r-process path with closed neutron shells have higher abundances than other isotopes.

Neutron stars (NS)

- Supported by neutron-degeneracy pressure.
- Decay of free neutrons is blocked by lack of available energy levels for electron from the decay (electrons become degenerate long before neutrons).
- $M_{\mathrm{NS}} < 2\text{-}3\ \mathrm{M_{Sun}}$
- Upper limit difficult to calculate; need general relativity.
- $R_{\mathrm{NS}} \sim 10$ km
- Small radius of NS compared to WD reflects the smaller de Broglie wavelength (greater mass) of the degenerate particle that supports the NS.
  - Small contribution due to degenerate electrons being outnumbered by nucleons in a WD, by a factor of 2:1.
- Rapid spin due to conservation of angular momentum L:

  $L = I\omega$ I = moment of inertia ω = angular speed

  leads to spin periods as short as milliseconds.
- Magnetic field strong $B_{\mathrm{NS}} \sim 10^8$ T due to its concentration by the core collapse (cf. $B_{\mathrm{Earth}} \sim 10^{-3}$ T).
- A Pulsar is a neutron star whose magnetic axis is not aligned with rotation axis, so spin of star modulates emission affected by magnetic field, e.g. radio, optical and X-ray.
- Spin rate of pulsar sets an upper limit on the radius (why?), and hence lower limit on the density for it to avoid break-up.
  - Equating gravitational and centrifugal acceleration:

    $$\langle\rho\rangle > 3\pi/G\tau_{\mathrm{obs}}^2$$
  - This confirms that short-period pulsars have mean densities well in excess of core densities of WDs.
  - Certainly comprised of matter at nuclear densities.

Black holes

If core mass exceeds limit supportable by neutron-degeneracy pressure, core will collapse further to a black hole.

Gravitational field is so strong that light cannot escape.

Full treatment requires general relativity, but can make some reasonable Newtonian approximations.

Escape velocity of a body is $v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$

Setting this equal to speed of light, c, gives $R = 2GM/\mathrm{c}^2$, called the Schwarzschild radius, R_{s}.

Collisions between infalling matter heat it to ~10^6 K, so see strong X-ray emission from vicinity of black holes.